

# Combined update scheme in the Sznajd model

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## Abstract

We analyze the Sznajd opinion formation model, where a pair of neighboring individuals sharing the same opinion on a square lattice convince its six neighbors to adopt their opinions, when a fraction of the individuals is updated according to the usual random sequential updating rule (asynchronous updating), and the other fraction, the simultaneous updating (synchronous updating). This combined updating scheme provides that the bigger the synchronous frequency becomes, the more difficult the system reaches a consensus. Moreover, in the thermodynamic limit, the system needs only a small fraction of individuals following a different kind of updating rules to present a non-consensus state as a final state.

*Key words:* Socio-physics, Opinion dynamics, Computer simulation.

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## 1 Introduction

The Sznajd model [1] is one of several recent consensus-finding models [2], in which each randomly selected pair of nearest neighbors convinces all its neighbors of the pair opinion, only if the pair shares the same opinion: “*United we stand, divided we fall*”. One find that starting with a random initial distribution of opinions, for a longer time and large systems, always a consensus is reached as the final state: everybody has the same opinion. It differs from other consensus models by dealing only with communication between neighbors, and the information flows outward, as in rumors spreading [3]. In contrast, in most other models the information flows inward, for instance the majority model

[4] and bootstrap percolation [5]. One of the reasons of its success [6] is its deep relationship with spin model like Ising. The similarities between Ising and Sznajd model lay, for instance, in the coarsening process [7], the scaling law for clusters growth [2] and persistence exponents [8]

Initially, two opinions ( $\pm 1$ ) are randomly distributed with probability  $p$  over all the nodes of the lattice, i.e., a fraction  $p$  of agents sharing the opinion  $+1$  and the rest of the agents having opinion  $-1$ . If the system evolves via a random sequential updating mechanism (asynchronously), as well as in the absence of perturbing factors like noise, always a phase transition is observed as a function of the the initial concentration  $p$ : for  $p < 0.5$  ( $p > 0.5$ ) all agents end up with opinion  $-1$  ( $+1$ ). However, if the asynchronous updating is replaced by a synchronous one, the possibility of reaching a full consensus is reduced quite dramatically [9,10].

The two-dimensional Sznajd model[11] is built on a square lattice with size  $L \times L = N$ . Each site  $s_{ij}$  ( $i, j = 1, 2, \dots, L$ ) is considered to be an individual, who can take one of the two possible opinions,  $s_{ij} = +1$  (positive opinion) or  $s_{ij} = -1$  (negative opinion). A pair of nearest neighbors convinces its six nearest neighbors of the pair's opinion if and only if both members of the pair have the same opinion; otherwise, the pair and its neighbors do not change their opinions. According to the system's updating schemes, it has, thus far, been always studied separately and performed in the following manner:

**Asynchronous Updating way:** One of the  $N$  sites in the square lattice is randomly selected as the first member of a pair, and then the other one is selected from its four neighbors. If and only if both members of the pair share the same opinion, they convince their six neighbors to adopt their opinion. The process continues until each of the  $N$  sites is selected once as the first member of the neighbor pair and it constitutes one time step.

**Synchronous Updating way:** Now, we go through the lattice like a type-writer to find the first member of a neighbor pair, and then we randomly choose the second member of the pair from the four neighbors of the first one. The opinions of its six neighbors at time step  $t + 1$  are updated taking into account the pair's opinion at time step  $t$ , i.e., if at time step  $t$  the pair has the same opinion, then only at time step  $t + 1$  its six neighbors will adopt the pair's opinion. Going through the whole lattice once constitutes one time step.

However, as we mentioned before, the Sznajd model when under synchronous updating reaches rarely the non-realistic full consensus observed when it evolves using the asynchronous rule. Therefore, one can argue that, as well as in the magnetic models, the synchronous updating can lead the system to much richer and different physics [12]. In this way, it seems to be interesting to analyze the updating effects on the system, moreover, when both updating rules are put

together, which could be performed if now we consider that a fraction of the individuals updates asynchronously and the remaining, synchronously. This is the aim of our paper: to understand more carefully the role of the updating mechanism on the Sznajd opinion formation model, when the system evolves partly synchronous and partly asynchronous.

## 2 Partly Synchronous and Partly Asynchronous Updating

In the beginning of the simulation, the system is divided into two classes:  $\alpha$  and  $\beta$ , i.e., at the initial time the individuals are randomly sorted into these two classes and they keep such classification during the whole simulation.  $N_f$  individuals  $\alpha$  and  $N - N_f$   $\beta$  ones. The neighbors of the  $\alpha$ -agents update their opinion synchronously, and the  $\beta$ -agents' neighbors, asynchronously. At every time step ( $t > 0$ ), we visit all the lattice nodes performing the synchronous updating at first and in a typewriter manner, then later the asynchronous one using a random sequence of individuals. The updating implementation is carried out according to the following steps:

- *Step 1:* Take an  $\alpha$ -agent as a first member of the pair, and choose one from its four neighbors to be the second member of the pair;
- *Step 2:* If the pair has the same opinion, then each of their six neighbors stores the pair opinion in a virtual memory (the purpose of this imaginary memory is to store all the opinions that an individual was persuaded to accept by some convincing neighbor pair); Otherwise, nothing happens.
- *Step 3:* Repeat *Step 1* and *Step 2* until all the  $\alpha$ -agents have been once selected as the first member of the pair.
- *Step 4:* Update the opinions of all individuals that have some opinions stored in their virtual memory (below we discuss this case in more details);
- *Step 5:* Take an  $\beta$ -agent as a first member of the pair, and choose one from its four neighbors to be the second member of a pair;
- *Step 6:* If the pair has the same opinion, then all their six neighbors adopt now the pair opinion; Otherwise, nothing happens.
- *Step 7:* Repeat *Step 5* and *Step 6* until all the  $\beta$ -agents have been once selected as the first member of the pair.

In both synchronous as well as asynchronous updating, an individual can belong to the neighborhoods of several convincing neighbor pairs. In the random sequential updating, the agent follows each neighbor pair in the order in which it gets the instruction to adopt the pair opinion. On the other hand, in the simultaneous updating (*Steps 1-4*), once an individual first collects from all its neighbor convincing pairs the order (which is stored temporarily in its virtual memory) to adopt their opinion, before updating its own opinion, then it does not know what to do if one neighbor pair orders to adopt opinion +1

and another also neighbor pair orders an opposite opinion. Thus, the individual should follow simultaneously two contradicting opinions. We then say it feels frustrated. In addition, this *frustration* [9,10,13] may occur also when the individual is selected as a member of a convincing pair and persuades its neighbors to adopt its opinion, but at the same time the same individual has been persuaded by its neighbor pair to adopt an opposite opinion (for instance, it persuades its neighbors to adopt the opinion  $+1$ , but it is persuaded to adopt the opinion  $-1$ ). In case of frustration, the agent does nothing, i.e., it keeps its own opinion, which hinders the system in reaching the full consensus state.

### 3 Simulation Results

In our simulations,  $p = 0.5$  (the initial concentration  $p$  of opinion  $+1$ ) and results are averaged over 1000 samples for lattice sizes  $L \leq 100$ , 100 samples for  $L > 100$  and after  $10^6$  Monte Carlo (MC) steps.

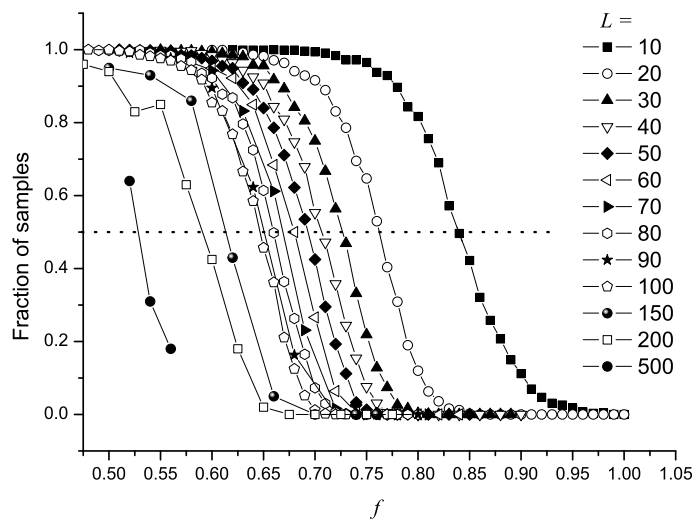


Fig. 1. Fraction of samples that reached a full consensus versus the frequency  $f$ .

Figure 1 shows how the fraction of samples that reached consensus varies with the frequency  $f = N_f/N$  of  $\alpha$ -agents and the lattice size  $L$ . As we can see, the bigger the synchronous frequency  $f$  becomes, the much more difficult the system reaches a consensus. This effect can be also observed for the same synchronous frequency  $f$  if the system size  $L$  increases. In Figure 2, we present how the frequency  $f$  value needed to get a consensus in half of the cases varies for different lattice sizes  $L$ . One observes that the frequency  $f$  varies roughly as  $L^{-0.1}$  and, moreover, if this power law holds up as  $L \rightarrow \infty$ , it means that any

nonzero and small positive value  $f$  always leads the system to a non-consensus as a final state.

For frequency  $f = 1$ , always frustration prevents consensus. However, for frequency  $f = 0$  and for  $f > 0$  considering that frustrated agents, instead of keeping their own opinions, always change their opinions, then always a complete consensus is found. In this case, to allow the frustrated individual to adopt the last persuaded opinion or the most probable one stored in its virtual memory leads always the system to a full consensus state, as one can observe in the traditional Sznajd model by taking into account the random sequential updating [11]. Both strategies direct the individual always to change its opinion, thus the formation of isolated domains of opinions into larger opposite opinion cluster is always prevented. When the system evolves only under synchronous updating, these domains could be also avoided and a full consensus could be reached, when introducing memory of past opinions, a frustrated individual changed its opinion to its most likely past opinion, as well as when every individual had an additional small probability for changing its opinion to a random one [9].

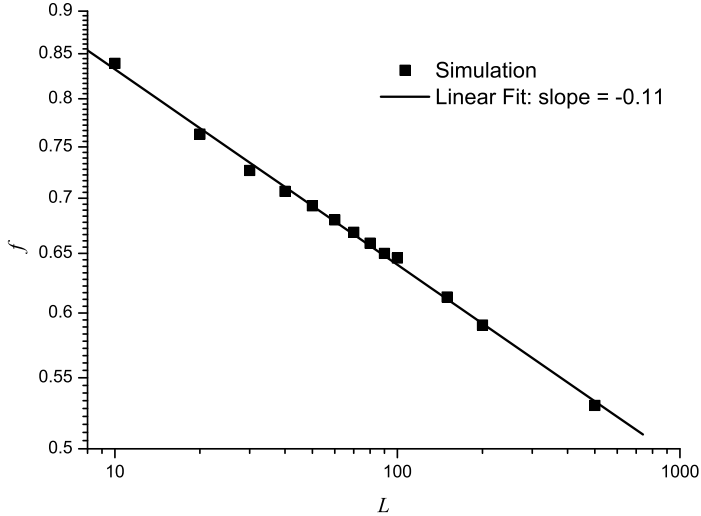


Fig. 2. Power-law relationship between the frequency  $f$  needed to get a consensus in half of the cases and the system size  $L$ . Both axes are logarithmic.

In order to investigate carefully a possible existence of phase transition for the mean opinion of the system versus the frequency  $f$ , we have calculated the mean opinion of the system, that corresponds the mean system magnetization and it is defined as:

$$m = \frac{1}{N} \sum_{i=1}^N s_i \quad (1)$$

where  $s_i = \pm 1$  is the individual opinion. Now we analyze our model as a function of  $f$ , in the same way in which Ising models are traditionally analyzed as a function of temperature  $T$ .

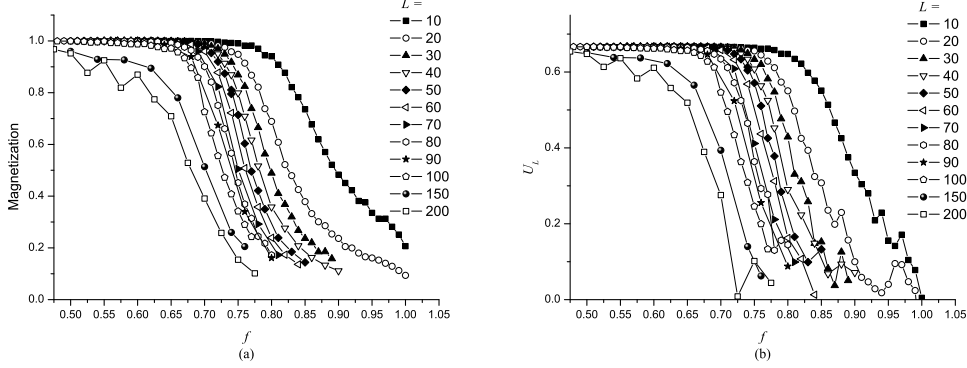


Fig. 3. Mean magnetization  $m$  (a) and Binder's cumulant  $U_L$  (b) as a function of the frequency  $f$  and for different system sizes  $L$ .

Because for small systems below the critical temperature  $T_c$ , the system often switches between positively and negatively states, the long-time averaged magnetization will be zero in these systems, which is clearly wrong. Then, in order to characterize more concretely phase transitions, one usually uses the Binder fourth-order magnetization cumulant  $U_L$  crossing technique [14]. This quantity  $U_L$  is expected, for sufficiently large systems, to present a unique intersection point when plotted versus the temperature  $t$  (frequency  $f$ ) for different choices of system sizes  $L$ . Moreover, the value of the temperature where this occurs is the value of the critical temperature  $T_c$  ( $f_c$ ).

$$U_L = 1 - \frac{\langle m^4 \rangle}{3 \langle m^2 \rangle^2} \quad (2)$$

where  $\langle \dots \rangle$  represent the thermal average (taken over the  $5 \times 10^5$  MC steps after discarding prior  $5 \times 10^5$  MC steps, and over all the samples).

Figure 3 presents the mean magnetization  $m$  (Fig. 3a) and the Binder's cumulant  $U_L$  (Fig. 3b) as a function of the frequency  $f$ . It can be observed that both  $m$  and  $U_L$  decrease when the system size increases. Besides, the absence of a unique crossing point in Binder's cumulant indicates the non-existence of phase transition at a finite frequency  $f$ . Furthermore, in the thermodynamic limit, i.e., for larger system and time steps, any nonzero fraction  $f$  of individuals following a different kind of updating rule always leads the system to a non-consensus as a final state, considering the same initial concentration of individuals with opinion  $+1$  and  $-1$ .

From Figure 4, we can notice the formation of opinion clusters inside of larger

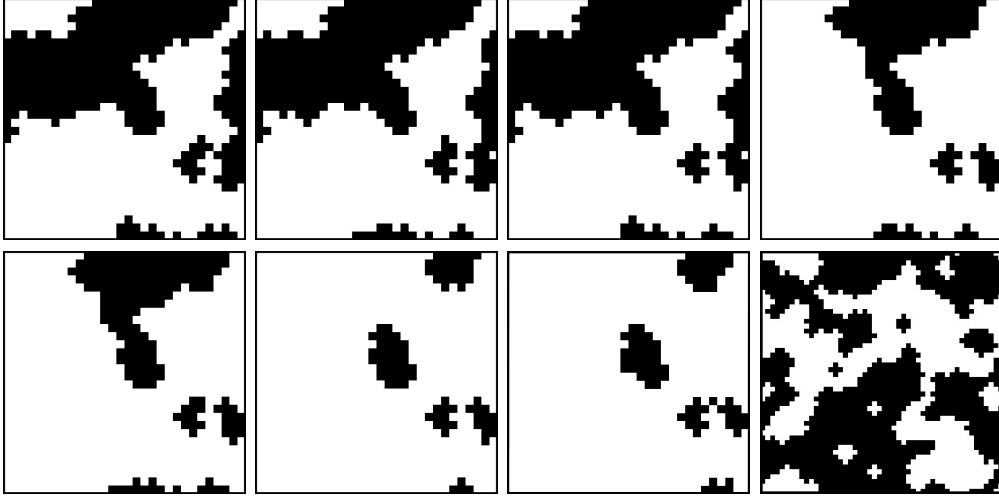


Fig. 4. Spatial distribution of opinions. The black (white) points correspond to individuals with opinion  $s_i = +1$  ( $s_i = -1$ ). For  $L = 30$ ,  $f = 0.8$  and different time steps  $t = 50128, 50361, 60074, 90058, 101284, 6014236, 2514897$ , from left to the right, top to bottom. The last one on the second row, which shows a very clear isolated white cross-like cluster inside a black cluster, is for an intermediate time step from a different simulation run.

opposite opinion clusters. Since the individual opinion stops changing its opinion when it feels frustrated, then this effect induces opposite opinion islands formation into the system, thus always there will exist dissidents in the system. In addition, once these islands emerge, many of them keep existing during all the simulation, i.e., they never disappear.

In summary, the assumption of combining two different updating schemes, asynchronous and synchronous, taken into account here comes from the urge of making the Sznajd model a bit more realistic, in the sense that the simultaneous updating could be imagined as the formal meetings at times fixed for all individuals, while the random sequential updating corresponds to the informal meetings of subgroups at various times. In addition, the results of our simulations in fact seem to be a convincing support for the claim that only a very small initial fraction of individuals following a different kind of updating is necessary to prevent the usual unrealistic complete consensus found for the traditional Sznajd model. More precisely, any nonzero fraction of formal meetings, that introduces frustration into the model, avoids all the individuals to have the same opinion.

## 4 Conclusion

In our paper, we introduce into the Sznajd model a combined updating mechanism, thus at a certain time some individuals change simultaneously their

opinion once, and others change separately their opinions several times: synchronous and asynchronous rules. The former could be interpreted as the formal meetings at times fixed for everybody, while the latter would be the informal meetings of subgroups at various times. Based on our results, one can conclude that never a complete consensus can be reached if there exists any formal meeting, in which the individuals are all together and, for instance, an individual is persuaded by all them at the same time to adopt their opinion, even contracting ones. Moreover, the consensus is always observed when the individual's decision or its persuasion occurs during the informal meetings of smaller several subgroups, the individual and a neighbor pair. The simultaneous updating causes frustration - when an agent should follow two opposite opinions at the same time. This effect could be clearly noticed through the formation of isolated domains of opinions into larger opposite opinion cluster. These islands of isolated opinions are kept existing during all the time evolution, i.e., always it would exist a fraction of dissidents in the population blocking consensus. Finally, the frequency  $f$  of individuals following a different kind of updating rule to provide a non-consensus state as a final state could be characterized by a power-law,  $f \sim L^{-0.1}$ .

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